Test 1A, MTH 1410

Name: _____

ID Number: _____

I pledge that I have neither given nor received any unauthorized assistance on this exam.

_____

(*signature*)

DIRECTIONS

1. Show all of your work. A correct answer with insufficient work will lose points, as will an answer with incorrect notation.
2. Clearly indicate your answer by putting a box around it.
3. Calculators are allowed on this exam.
4. Give all answers in exact form, not decimal form (that is, put π instead of 3.1415, $\sqrt{2}$ instead of 1.414, etc) unless otherwise stated.
5. Make sure you sign the pledge and write your ID on both pages.
6. Number of questions = 8. Total Points = 100.

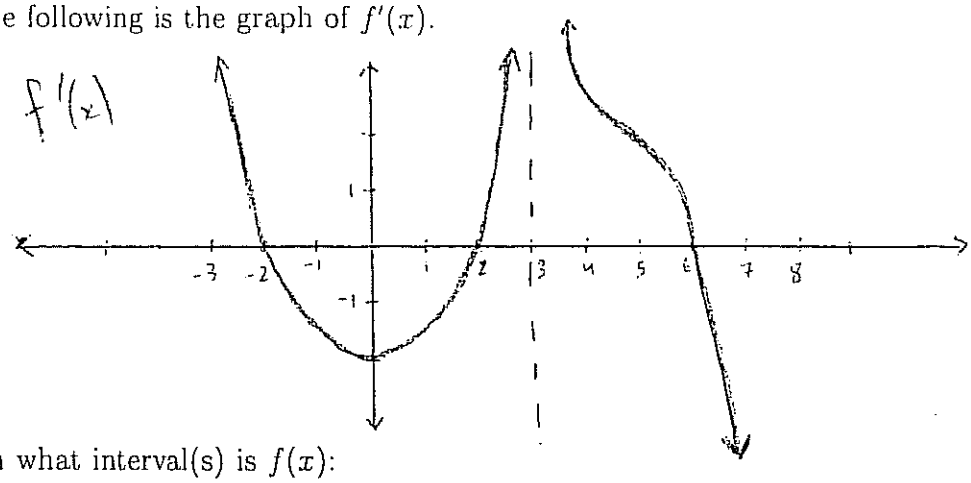
ID Number: _____

1. (16 points) Use the limit definition of the derivative to calculate $f'(x)$ if

$$f(x) = \frac{3}{x^2}.$$

Make sure you use correct notation.

2. (9 points) For this question, pay close attention to which one is f and which one is f'. The following is the graph of $f'(x)$.



On what interval(s) is $f(x)$:

(a) Increasing

(b) Decreasing

(c) Concave down

3. (6 points) Is the statement below true or false? If true, briefly explain why. If false, give a counterexample or explain your reasoning.

(a) " If $\lim_{x \to 3} f(x) = f(3)$, then $f'(3)$ exists. "

4. (16 points) Use the derivative laws (i.e. not the definition) to calculate the equation of the tangent line to

$$f(x) = \frac{x^3 + 2\sqrt{x^3} - 7}{x}$$

at $x = 1$.

5. (15 points) Find all horizontal and vertical asymptotes of

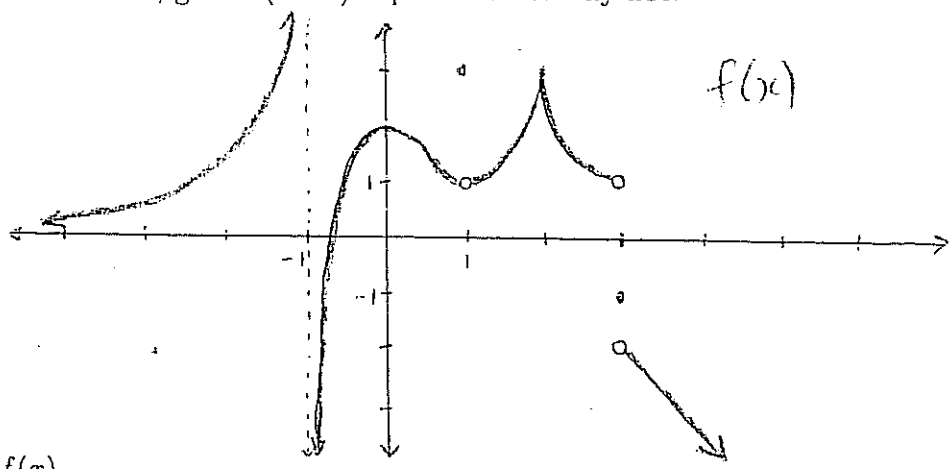
$$\frac{3x^2 - 3x - 60}{x^2 - 16}.$$

Explain your answer, and make sure to use correct notation.

6. (16 points) Find the value of c which will make the following function continuous at $x = 9$. Make sure to use correct notation and explain your reasoning:

$$f(x) = \begin{cases} \left[\dfrac{\sqrt{x} - 3}{x - 9}\right] & \text{if } x > 9 \\ 2c & \text{if } x \leq 9 \end{cases}$$

7. (10 points) For the given graph, find the value for the expressions below. If the expression does not exist, give a (brief) explanation of why not.



(i) $\lim_{x \to (-1)^-} f(x)$

(ii) $\lim_{x \to 2^+} f(x)$

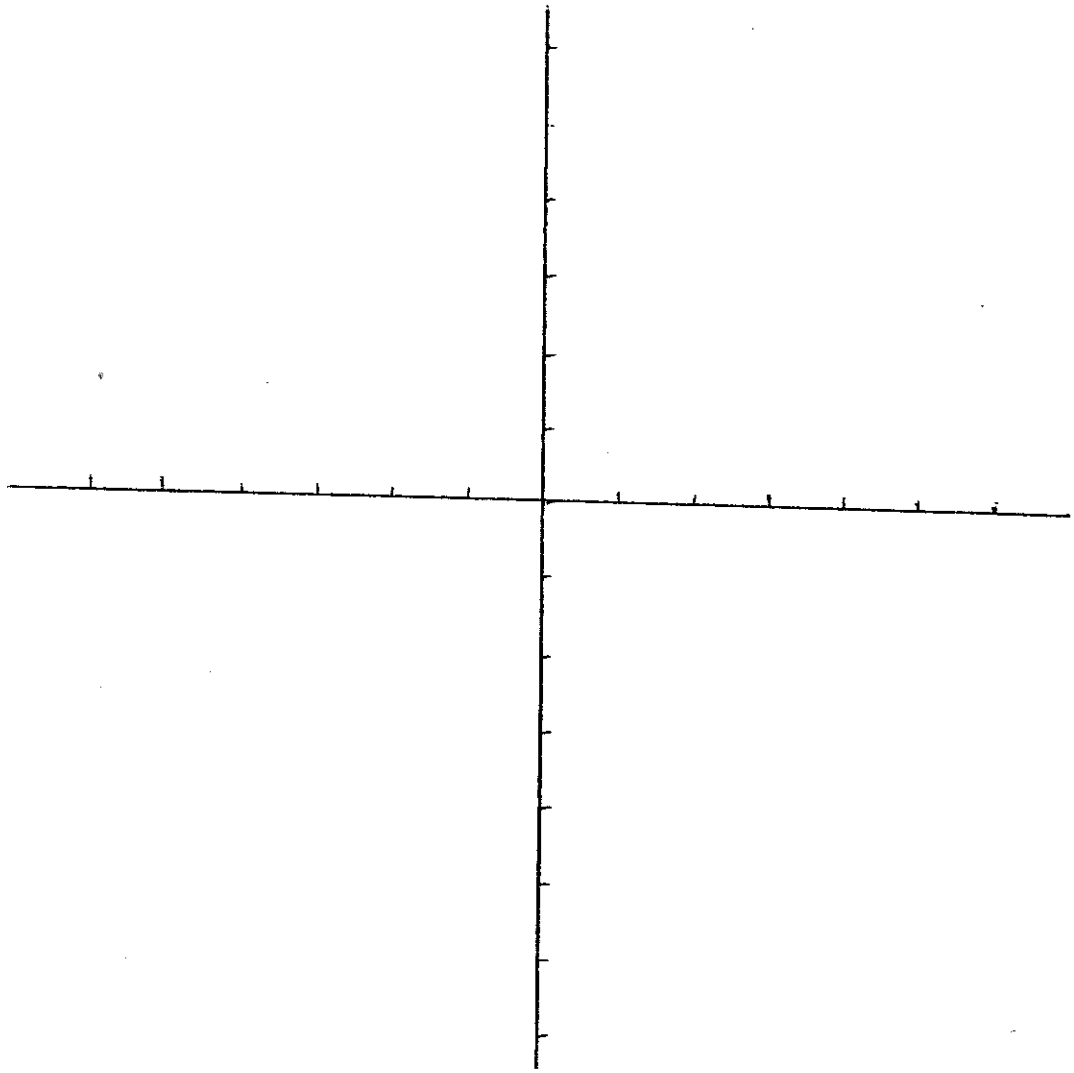
(iii) $\lim_{x \to (-\infty)} f(x)$

(iv) $\lim_{x \to 3} f(x)$

(v) $f'(1)$

8. (12 points) Sketch the graph of a continuous function $f(x)$ that satisfies the following conditions. Label all local maximums, local minimums, and inflection points on your graph:

- $f(-2) = -3$ and $\lim_{x \to 2} f(x) = 4$
- $f'(-2) = 0$ and $f'(2) = 0$
- $f'(x) < 0$ on the intervals $(-\infty, -2)$ and $(2, \infty)$; $f'(x) > 0$ on the interval $(-2, 2)$.
- $f''(x) < 0$ on the intervals $(-\infty, -3)$ and $(0, \infty)$; $f''(x) > 0$ on the interval $(-3, 0)$
- $\lim_{x \to -\infty} f(x) = 1$



Extra Credit(2 points) Use the derivative rules (not the limit definition) to find $f'(x)$ if $f(x) = e^7$